# Разработка параллельных алгоритмов для современных вычислительных систем – матанализ вместо арифметики

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Летняя Суперкомпьютерная Академия 1 июля 2016



#### Outline

Mainstream Performance Optimization Algorithms

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Application performance profiles on modern platforms

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Application performance profiles on modern platforms

Challenges and Benefits of Accepting Reality Experimental Results

#### Mainstream approach to performance modelling

Most of algorithms for performance optimization are based on very simple models:

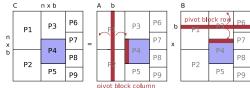
- ► Scheduling algorithms
- ► Load balancing algorithms
- ▶ Data partitioning algorithms
- ► Task mapping algorithms

They assume the speed of processing element to be constant.

#### Matrix partitioning

Matrix partitioning problem for parallel matrix multiplication on heterogeneous platforms\*

- ► Input: constant processor speeds
- ► Matrices partitioned so that
  - ► Area of the rectangle proportional to the speed
  - ▶ Volume of communication minimized



Maths used by algorithms solving this problem do not go beyond basic arithmetics.

<sup>\*</sup> Beaumont, O. et al: Matrix Multiplication on Heterogeneous Platforms. IEEE Trans. Parallel Distrib. Syst. 2001

#### Traditional Dynamic Load Balancing Algorithms

A routine has n computational units distributed across p processors.

Processor  $P_i$  has  $d_i$  units such that  $n = \sum_{i=1}^p d_i$  Initially  $d_i^0 = n/p$ 

At each iteration

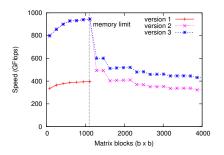
- 1. Execution times for this iteration measured and gathered to root
- 2. if relative difference between times  $\leq \epsilon$  then no balancing needed else new distribution is calculated as:  $d_i^{k+1} = n \times s_i^k / \sum_{j=1}^p s_j^k \quad \text{where speed } s_i^k = d_i^k / t_i (d_i^k)$
- 3. new distributions d<sub>i</sub><sup>k+1</sup> broadcast to all processors and where necessary data is redistributed accordingly.

#### Domain decomposition in CFD

Parallel CFD packages such as OpenFOAM use graph/mesh partitioning libraries for domain decomposition

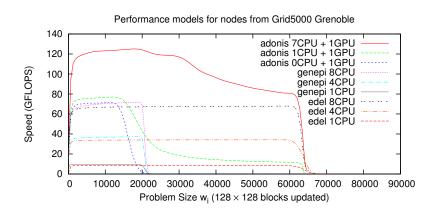
- ► MeTiS, Scotch, etc.
- ▶ Input vector of positive constants representing the relative volume of computation to be performed by each processor

#### Functional Performance Models of GPU

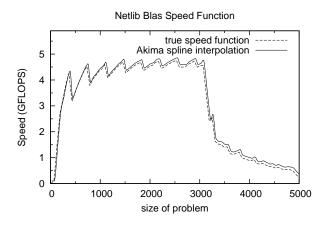


- $\triangleright$  g(x) (version 1): naive kernel
- ▶ g(x) (version 2): accumulate intermediate result + out-of-core
- ► g(x) (version 3): version 2 + overlap data transfers and kernel executions

#### Experimental results for Grid'5000 nodes

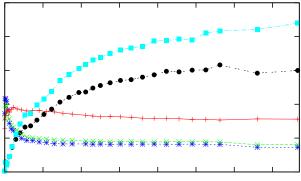


Experimental results for Netlib BLAS dgemm\*

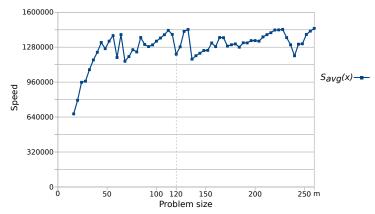


<sup>\*</sup> Rychkov, V. et al: Using Multidimensional Solvers for Optimal Data Partitioning on Dedicated Heterogeneous HPC Platforms. PaCT'2011

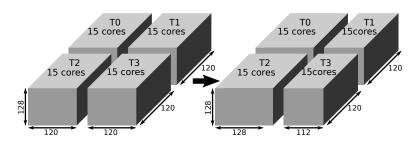
Speed functions of the CG solver built in different configurations on an Adonis node (GFlops against the number of control volumes)



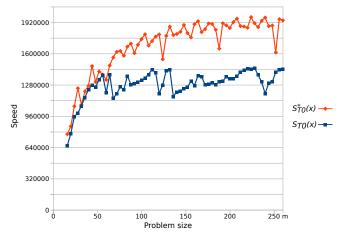
Experimental results for MPDATA on Intel Xeon Phi (domain size  $120 \times m \times 128$ )



Experimental results for MPDATA on Intel Xeon Phi

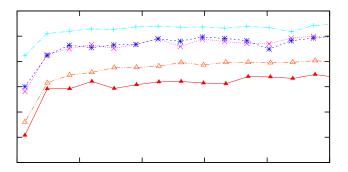


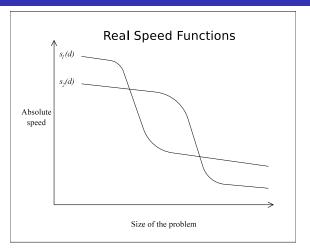
More experimental results for MPDATA on Intel Xeon Phi: the impact of resource sharing



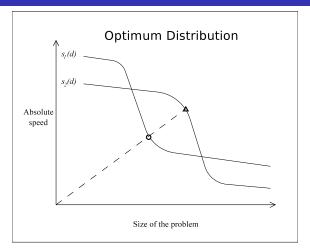
The impact of resource sharing: the speed of a CPU core built in different configurations

ightharpoonup s<sub>1</sub>(x), s<sub>6</sub>(x), s<sub>12</sub>(x), S<sub>6</sub>(6x)/6, S<sub>12</sub>(12x)/12

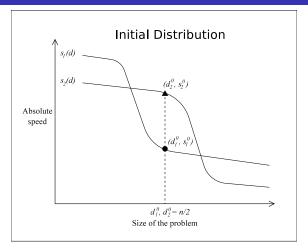




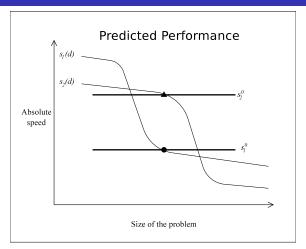
<sup>\*</sup> Clarke, D. et al: Dynamic Load Balancing of Parallel Computational Iterative Routines on Highly Heterogeneous HPC Platforms. Parallel Processing Letters, 2011



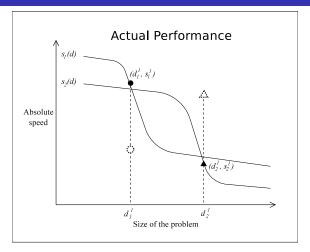
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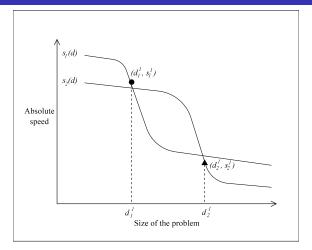
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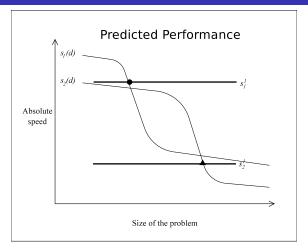
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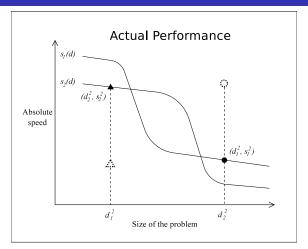
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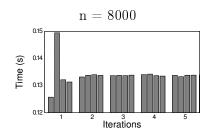
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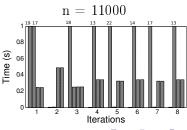
#### Iterative Routine

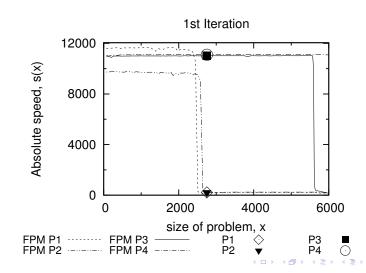
Jacobi method for solving a system of linear equations.

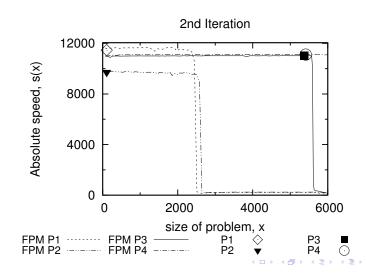
Experimental Setup

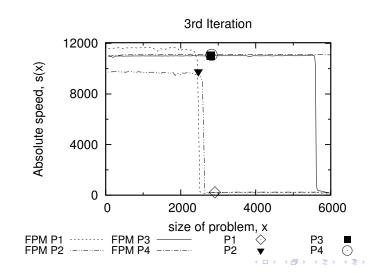
T.	I .			
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>
Processor	3.6 Xeon	3.0 Xeon	3.4 P4	3.4 Xeon
Ram (MB)	256	256	512	1024

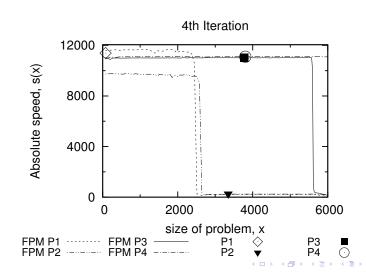


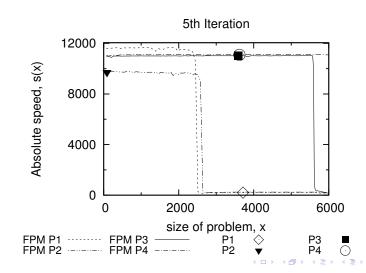












# Complex but realistic load balancing does always balance

Face the reality and use speed functions instead of constants in dynamic load balancing.

#### Challenges:

- ▶ Non-trivial partitioning algorithms manipulating by functions, not numbers.
- ▶ Non-trivial technique to build the speed functions suitable for the algorithms.

#### Benefits:

▶ Performance gains.

# FPM-based Dynamic Load Balancing Algorithm

- ▶ Algorithm is based on models for which speed is a function of problem size.
- ► Load balancing achieved when:

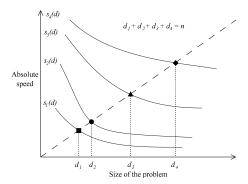
$$t_i \approx t_j, \quad 1 \le i, j \le p$$
 (1)

$$\frac{d_1}{s_1(d_1)} \approx \frac{d_2}{s_2(d_2)} \approx \dots \approx \frac{d_p}{s_p(d_p)}$$
 (2)

where 
$$d_1 + d_2 + \cdots + d_p = n$$

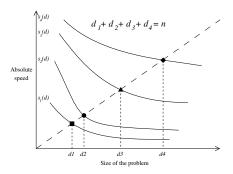
#### Solving Distribution Problem

▶ Problem is solved geometrically by noting that the points  $(d_i, s_i(d_i))$  lie on a line passing through the origin when  $\frac{d_i}{s_i(d_i)} = \text{constant}$ .



#### FPM-based data partitioning algorithm

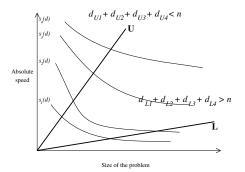
- ► Total problem size determines the slope
- ▶ Algorithm iteratively bisects solution space to find values d<sub>i</sub>



<sup>\*</sup> Lastovetsky, A. and Reddy, R.: Data Partitioning with a Functional Performance Model of Heterogeneous Processors. Int. J. of High Perf. Comp., App., 2007

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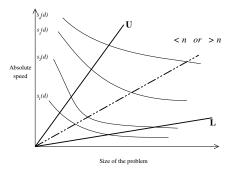
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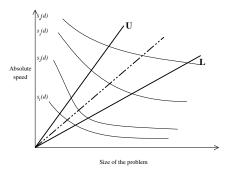
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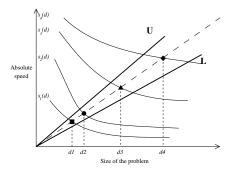
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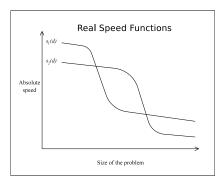
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Functional Performance Models may be built:

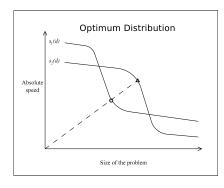
- exhaustively in advance
- dynamically at run time\*



<sup>\*</sup> Lastovetsky, A. and Reddy, R.: Distributed Data Partitioning for Heterogeneous Processors Based on Partial Estimation of their Functional Performance Models. HeteroPar'2009

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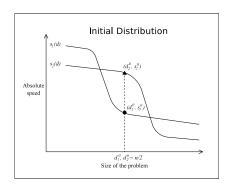


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# Functional Performance Models may be built:

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- ▶ dynamically at run time\*

Initial: point  $(n/p, s_i^0)$  with speed  $s_i^0 = \frac{n/p}{t_i(n/p)}$  first function approximation  $s_i'(x) \equiv s_i^0$ 

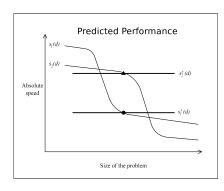


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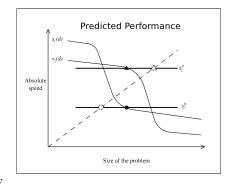
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 first function approximation 
$$s_i'(x) \equiv s_i^0$$

Iterations: point  $(d_i^k, s_i^k)$  with speed

$$\mathrm{s_i^k} = rac{\mathrm{d_i^k}}{\mathrm{t_i(d_i^k)}}$$

approximation  $s'_i(x)$  updated by

adding the point



Functional Performance Models may be built:

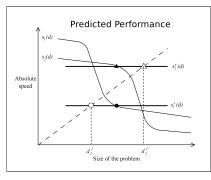
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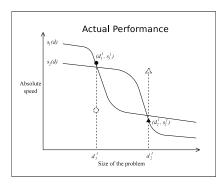
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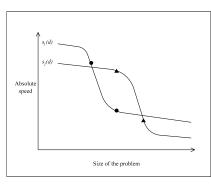
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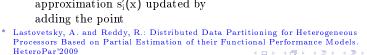
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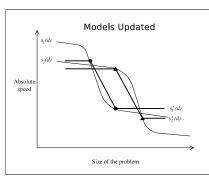
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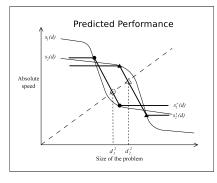
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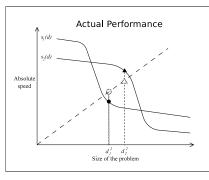
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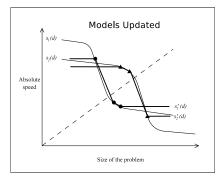
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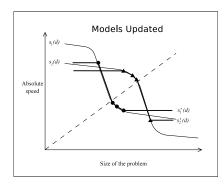
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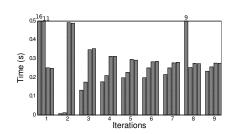
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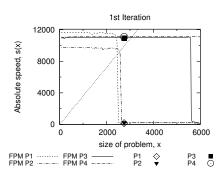
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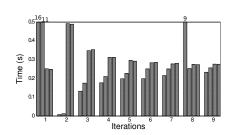
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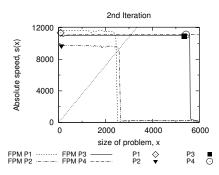
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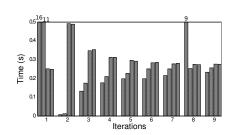


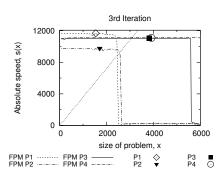


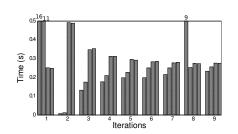


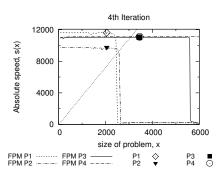


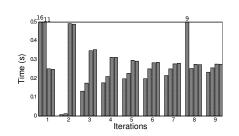


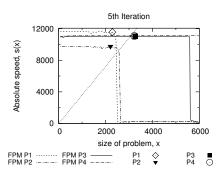


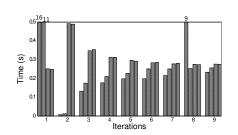


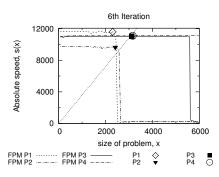


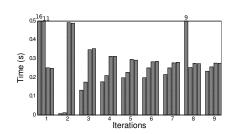


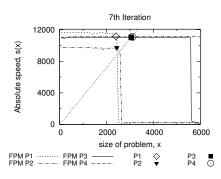


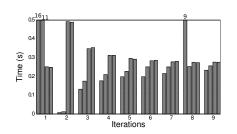


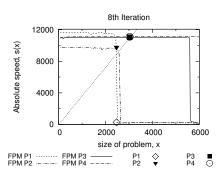


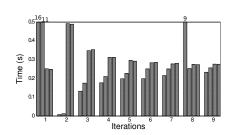


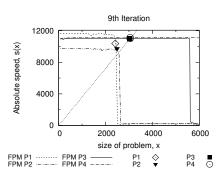






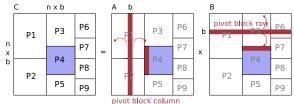






# Matrix Multiplication on Heterogeneous Platform\*

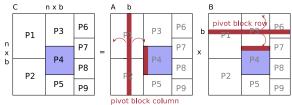
- ► Input: constant processor speeds
- ▶ Matrices partitioned so that
  - ► Area of the rectangle proportional to the speed
  - ► Volume of communication minimized



<sup>\*</sup> Beaumont, O. et al: Matrix Multiplication on Heterogeneous Platforms. IEEE Trans. Parallel Distrib. Syst. 2001

# Matrix Multiplication on Heterogeneous Platform\*

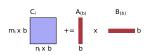
- ► Input: constant processor speeds
- ▶ Matrices partitioned so that
  - ▶ Area of the rectangle proportional to the speed
  - ► Volume of communication minimized



- ► More accurate solution is based on speed functions as input\*\*
- \* Beaumont, O. et al: Matrix Multiplication on Heterogeneous Platforms. IEEE Trans. Parallel Distrib. Syst. 2001
- \*\* Clarke, D. et al: Column-Based Matrix Partitioning for Parallel Matrix Multiplication on Heterogeneous Processors Based on Functional Performance Models. In: HeteroPar-2011, LNCS 7155, 2012

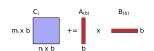
### Matrix Multiplication on Heterogeneous Platform

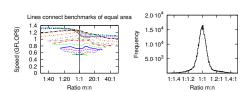
Computational kernel: panel-panel update



# Matrix Multiplication on Heterogeneous Platform

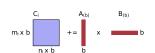
- ► Computational kernel: panel-panel update
- ► Processor speed function of area Built by running the kernel for square matrices

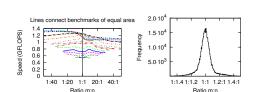


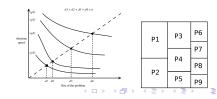


### Matrix Multiplication on Heterogeneous Platform

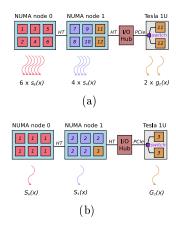
- Computational kernel: panel-panel update
- Processor speed function of area
   Built by running the kernel for square matrices
- ► FPM-based
  partitioning algorithm
  finds the optimal areas
  The areas are used as
  input to the matrix
  partitioning algorithm







#### Matrix multiply on hybrid node: performance modelling



<sup>\*</sup> Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

# Matrix multiplication on hybrid node

#### Experimental platform

	CPU (AMD)	GPUs (N	VIDIA)
Architecture	Opteron 8439SE	GF GTX680	Tesla C870
Core Clock	$2.8~\mathrm{GHz}$	$1006~\mathrm{MHz}$	$600~\mathrm{MHz}$
Number of Cores	$4 \times 6$ cores	1536  cores	128  cores
Memory Size	$4 \times 16 \text{ GB}$	$2048~\mathrm{MB}$	$1536~\mathrm{MB}$
Memory Bandwidth		$192.3~\mathrm{GB/s}$	$76.8~\mathrm{GB/s}$

<sup>\*</sup> Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

Execution time of the application under different configurations

Matrix size (blks)	CPUs (sec)	GTX680 (sec)	Hybrid-FPM (sec)
$40 \times 40$	99.5	74.2	26.6
$50 \times 50$	195.4	162.7	77.8
$60 \times 60$	300.1	316.8	114.4
$70 \times 70$	491.6	554.8	226.1

Column 1: block size is  $640 \times 640$ 

Column 2:  $4 \times 6$  CPU cores, homogeneous data partitioning

Column 3: CPU core + GPU

Column 4:  $2 \times 6$  CPU cores  $+ 2 \times 5$  CPU cores  $+ 2 \times ($  CPU core + GPU ),

<sup>\*</sup> Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

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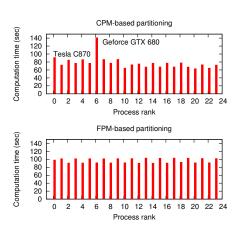
Column 2:  $4 \times 6$  CPU cores, homogeneous data partitioning

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Column 4:  $2 \times 6$  CPU cores  $+ 2 \times 5$  CPU cores  $+ 2 \times ($  CPU core + GPU ),

<sup>\*</sup> Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

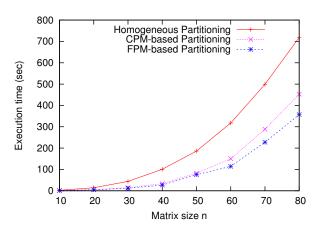
# Computation time of each process



#### Matrix size $60 \times 60$ , Computation time reduced by 40%

\* Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

### Performance with different partitionings

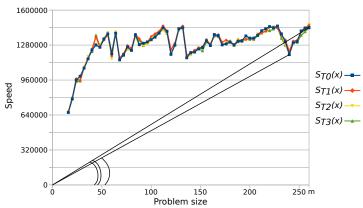


Execution time reduced by 23% and 45% respectively

\* Zhong, Z. et al.: Data Partitioning on Multicore and Multi-GPU Platforms Using Functional Performance Models. IEEE Transactions on Computers, 2015

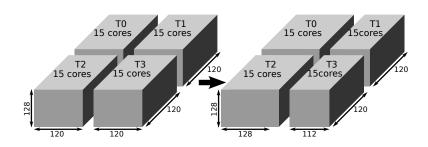
# Performance optimization through load imbalancing

MPDATA can be optimized by imbalancing the load of processors\*



<sup>\*</sup> Lastovetsky, A., Shustak, L., Wyrzykowski, R.: Model-based optimization of MPDATA on Intel Xeon Phi through load imbalancing. arxiv.org preprint, 2015

# Performance optimization through load imbalancing



# Summary

- ► Traditional algorithms are easy to design but not accurate.
- ► FPM-based algorithms are not trivial and require mathematical skill beyond arithmetics and discrete maths.
  - ▶ This is a big problem and CS curriculum does not teach how to apply non-discrete maths in the context of CS. CS students believe that math analysis is something that only physicists need.
- ▶ It is not easy to go outside the box but benefits are there...

# Спасибо за внимание!

Вопросы?

